Supplementary Material to "Dual Moments and Risk Attitudes"

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Abstract

This online appendix provides the proof of a result in Section 5 and two illustrations to supplement Section 7.

A Proof of Proposition 5.1

First, note that (i) is equivalent to

(iv) $U_2(U_1^{-1}(t))$ and $h_2(h_1^{-1}(u))$ are concave functions of t and u for all t and all $u \in (0,1)$.

(v) $\frac{U_2(y)-U_2(x)}{U_2(w)-U_2(v)} \leq \frac{U_1(y)-U_1(x)}{U_1(w)-U_1(v)}$ and $\frac{h_2(s)-h_2(r)}{h_2(q)-h_2(p)} \leq \frac{h_1(s)-h_1(r)}{h_1(q)-h_1(p)}$ for all $v < w \leq x < y$ and all $0 < p < q \leq r < s < 1$.

The equivalence of (i), (iv) and (v) follows trivially from the equivalence of (a), (d) and (e) in Theorem 1 of Pratt [1] and the corresponding DT counterpart equivalences.

Second, we will prove that (the equivalent) (i), (iv) and (v) imply (ii). Reconsider (4.1). Fix (a feasible) $\varepsilon_1 > 0$ (satisfying $0 < \varepsilon_1 \leq \{p_0, 1-p_0\} < 1$). Note that if we let $\varepsilon_2 \to 0$ in (4.1), then $\lambda_i \to 0$. Define

$$\begin{aligned} V_i(\lambda_i, \varepsilon_2) = & \left(h_i(p_0+\varepsilon_1) - h_i(p_0-\varepsilon_1)\right) U_i\left(w_0-\lambda_i\right) \\ & - \left(\left(h_i(p_0) - h_i(p_0-\varepsilon_1)\right) U_i(w_0-\varepsilon_2) + \left(h_i(p_0+\varepsilon_1) - h_i(p_0)\right) U_i(w_0+\varepsilon_2)\right). \end{aligned}$$

We compute the total differential $\mathrm{d}V_i = \frac{\partial V_i}{\partial \lambda_i}\,\mathrm{d}\lambda_i + \frac{\partial V_i}{\partial \varepsilon_2}\,\mathrm{d}\varepsilon_2$. It is given by

$$\begin{aligned} & -\left(h_i(p_0+\varepsilon_1) - h_i(p_0-\varepsilon_1)\right) U_i'\left(w_0-\lambda_i\right) \mathrm{d}\lambda_i \\ & + \big(\left(h_i(p_0) - h_i(p_0-\varepsilon_1)\right) U_i'(w_0-\varepsilon_2) - \left(h_i(p_0+\varepsilon_1) - h_i(p_0)\right) U_i'(w_0+\varepsilon_2)\big)\,\mathrm{d}\varepsilon_2. \end{aligned}$$

Equating the total differential to zero yields

$$\frac{\mathrm{d}\lambda_i}{\mathrm{d}\varepsilon_2} = \frac{h_i(p_0) - h_i(p_0-\varepsilon_1)}{h_i(p_0+\varepsilon_1) - h_i(p_0-\varepsilon_1)} \frac{U_i'(w_0-\varepsilon_2)}{U_i'(w_0-\lambda_i)} - \frac{h_i(p_0+\varepsilon_1) - h_i(p_0)}{h_i(p_0+\varepsilon_1) - h_i(p_0-\varepsilon_1)} \frac{U_i'(w_0+\varepsilon_2)}{U_i'(w_0-\lambda_i)}. \tag{A.1}$$

From (i), as in Pratt [1], Eqn. (20),

$$\frac{U_2'(x)}{U_2'(w)} \leq \frac{U_1'(x)}{U_1'(w)}, \qquad \text{for } w < x, \qquad \text{and} \qquad \frac{U_2'(x)}{U_2'(y)} \geq \frac{U_1'(x)}{U_1'(y)}, \qquad \text{for } x < y.$$

Furthermore, from (v),

$$\frac{U_2(y)-U_2(x)}{U_2(w)-U_2(v)} + \frac{U_2(w)-U_2(v)}{U_2(w)-U_2(v)} \leq \frac{U_1(y)-U_1(x)}{U_1(w)-U_1(v)} + \frac{U_1(w)-U_1(v)}{U_1(w)-U_1(v)}, \qquad \text{for } v < w \leq x < y.$$

Taking $w = x$ yields

$$\frac{U_2(y)-U_2(v)}{U_2(w)-U_2(v)} \leq \frac{U_1(y)-U_1(v)}{U_1(w)-U_1(v)}, \qquad \text{for } v < w < y,$$

hence

$$\frac{U_2(w) - U_2(v)}{U_2(y) - U_2(v)} \geq \frac{U_1(w) - U_1(v)}{U_1(y) - U_1(v)}, \quad \text{and also} \quad \frac{U_2(y) - U_2(w)}{U_2(y) - U_2(v)} \leq \frac{U_1(y) - U_1(w)}{U_1(y) - U_1(v)},$$

for $v < w < y$. In all inequalities in this paragraph, U_i may be replaced by h_i, with v, w, x and y restricted to $(0, 1)$.

Thus, from (A.1) and the inequalities above,

$$\frac{\mathrm{d}\lambda_2}{\mathrm{d}\varepsilon_2} \geq \frac{\mathrm{d}\lambda_1}{\mathrm{d}\varepsilon_2}, \tag{A.2}$$

hence (ii).

We have now proved that (ii) is implied by (the equivalent) (i), (iv) and (v). We finally show that (ii) implies (i), or rather that not (i) implies not (ii). This goes by realizing that, by the arbitrariness of w_0, p_0, ε_1 with $0 < \varepsilon_1 \leq \{p_0, 1 - p_0\} < 1$, and $\varepsilon_2 > 0$, if (i) does not hold on some interval (of w or p), one can always find feasible w_0, p_0, ε_1 and ε_2, such that (A.2), hence (ii), hold on some interval but with the inequality signs strict and flipped. □

B Additional Figures

Figure B.1: Tversky-Kahneman Probability Weighting Function (upper panel) and its Local Index (lower panel). See (7.3). We consider $\beta \in \{0.55, 0.65, \ldots, 0.95\}$.

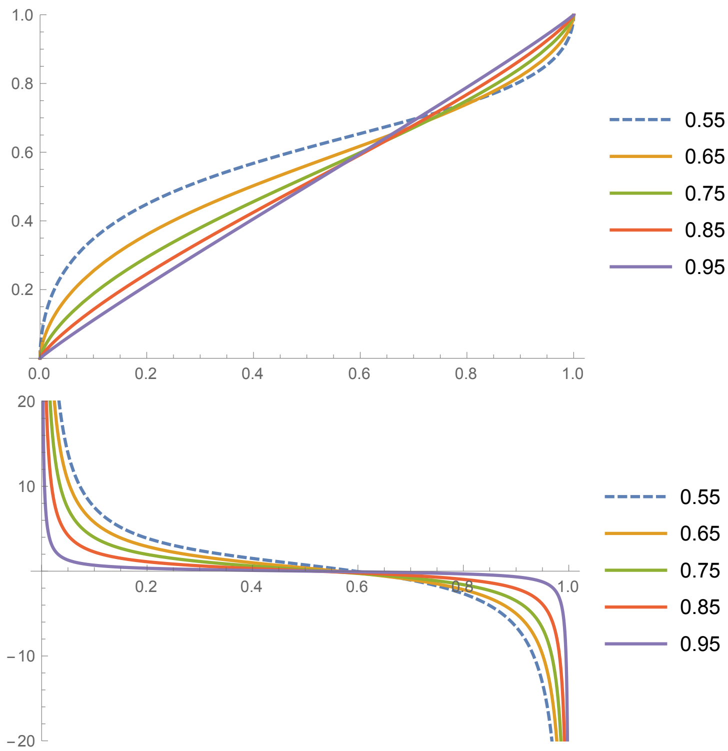
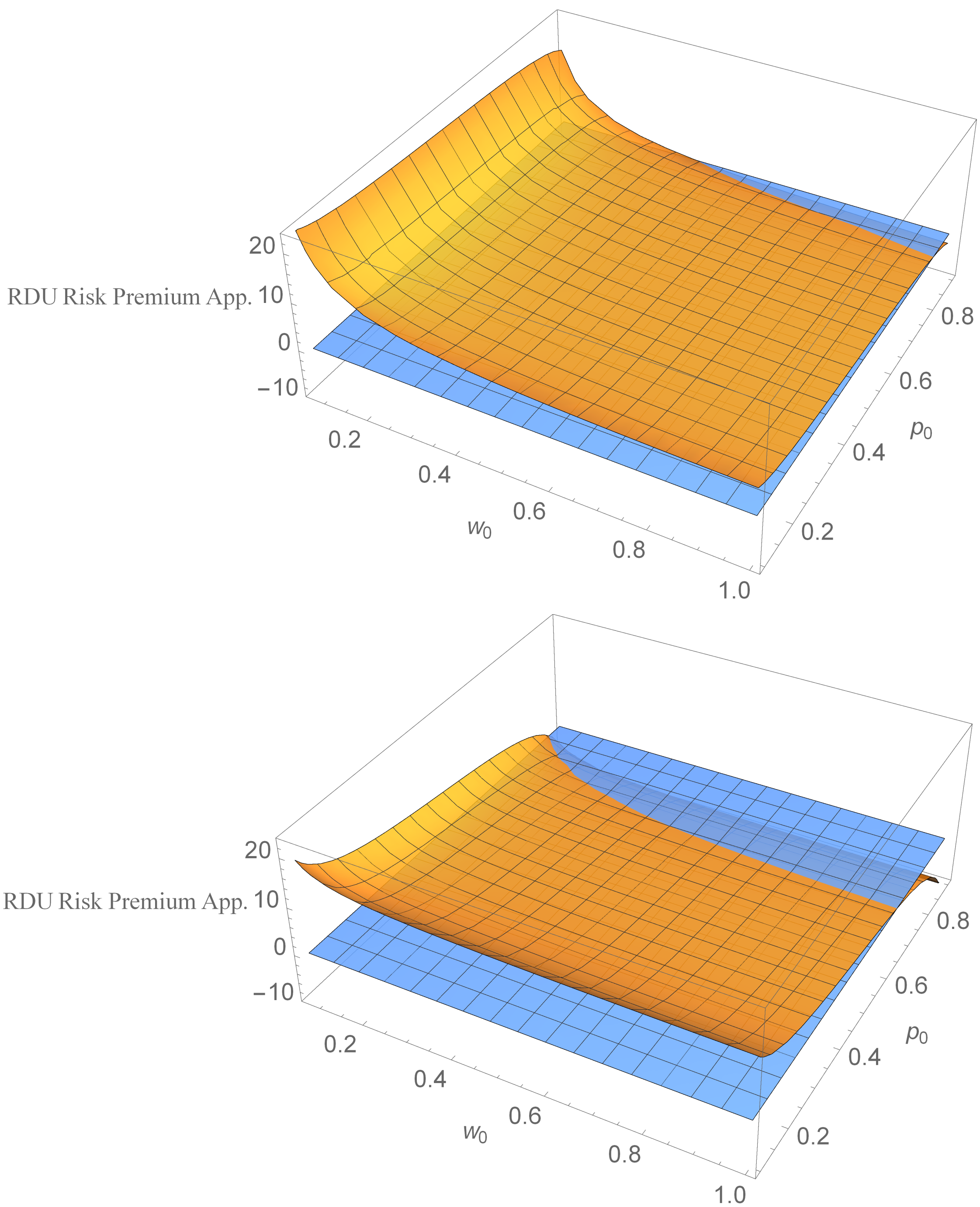


Figure B.2: Surface of the RDU Risk Premium Approximation (4.2). We consider a risk with small variance and maxiance, normalized to satisfy $\frac{\bar{\mathtt{m}}_2}{2\mathtt{Pr}} = 1$ with $\mathtt{m}_2/\bar{\mathtt{m}}_2 = 3$ (upper panel), and $\frac{\mathtt{m}_2}{2\mathtt{Pr}} = 1$ with $\mathtt{m}_2/\bar{\mathtt{m}}_2 = 1/3$ (lower panel), under power utility ((7.4) with $\gamma = 0.5$) and Prelec's probability weighting function ((7.1) with $\alpha = 0.65$). The orange surface represents our approximation (4.2) to the RDU risk premium λ, while the blue surface is the $\lambda = 0$-plane.



References

[1] Pratt, J.W. (1964). Risk aversion in the small and in the large. *Econometrica* 32, 122-136.